MONOPOLISTIC COMPETITION IN A LIMITED ORBITAL SPACE

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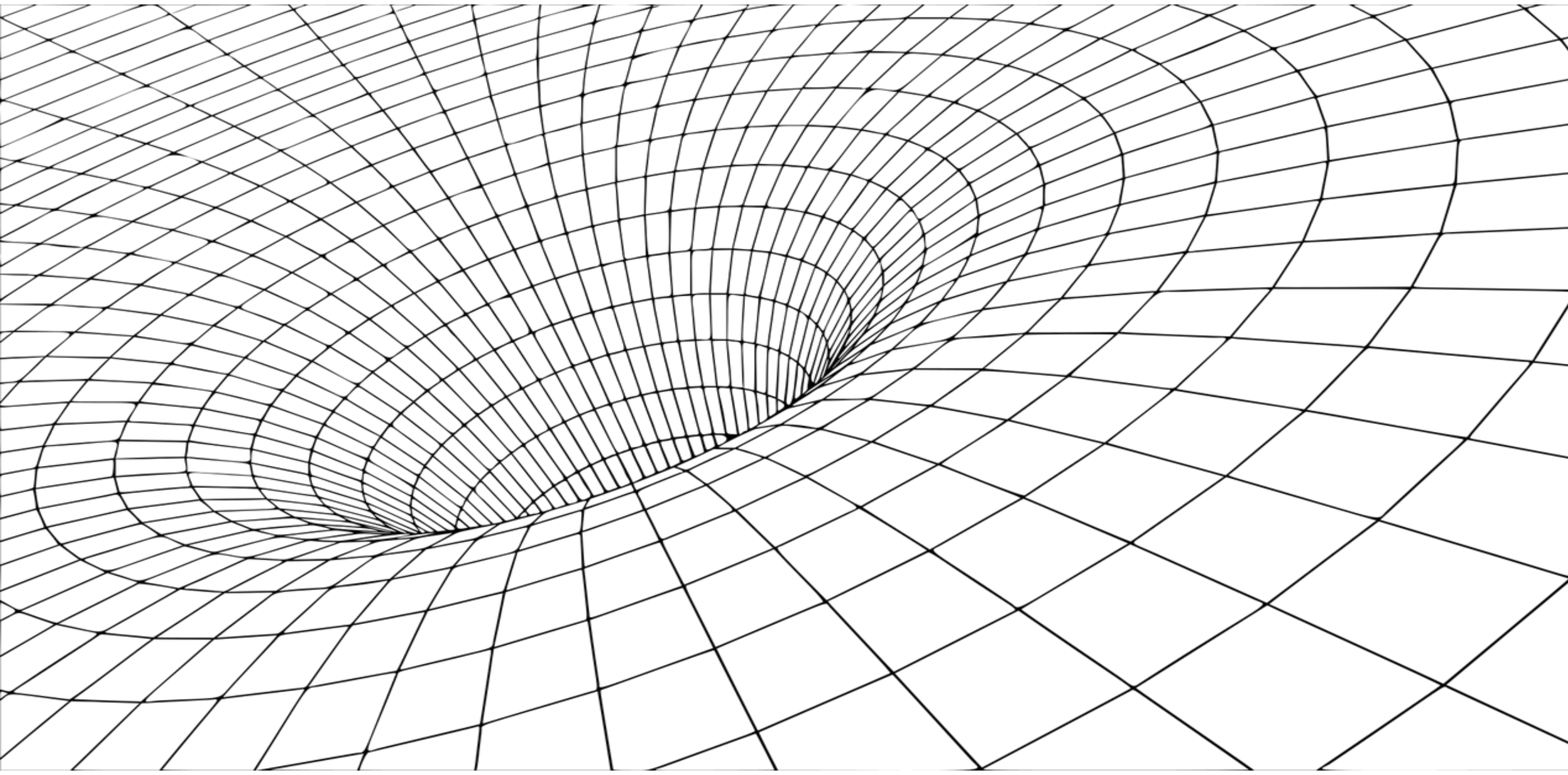

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Abstract

In a context of intense competition for access to the Earth's orbit, we study a model of monopolistic competition in which satellites operators diversify the variety of satellite services. We put this in perspective with the accumulation of in-orbit fragment debris and the risk it poses for the sustainability of orbital activity. Monopolistic competition leads to a sub-optimal outcome, in terms of both the number of satellites in orbit and the range of services offered. We show that monopolistic competition results in excessive orbit congestion, when Earth's orbit carrying capacity is low and/or consumers' preference for diversity is low, and always leads to an insufficient number of satellite services being offered. However, a strong consumers' preference for service diversity, as it increases the market power of satellites operators, can mitigate congestion of the Earth's orbit.

1 Introduction

The launch of the first artificial satellite, Sputnik, into orbit in 1957 demonstrated the technological feasibility of using the Earth's orbital space. Since then, the space industry has developed a wide range of practical applications for this technology, covering fields as diverse as Earth observation, positioning, meteorology, communication, television and the Internet. Although at various stages of development, new applications are expected in the near future, notably in the areas of advertising, broadband Internet, space-based solar panels and suborbital tourism.

The economic literature is ill-equipped to analyse the growing diversity of satellite services. Most often, it formalizes the population of satellites in orbit as a homogeneous set, supposedly offering consumers a composite service at a single price. With a few exceptions, it also assumes pure and perfect competition. These simplifying assumptions proved effective in creating an initial analytical framework, both representing the physics and the exploitation of the Earth's orbit by private satellite operators.

Using this basic analytical framework, the existing literature has shown that, in the absence of appropriate regulation, private satellite operators will tend to place too many artificial satellites in orbit (Adilov, Alexander and Cunningham, 2018; Rao, Burgess and Kaffine, 2020; Rouillon, 2020). This is because they will largely disregard the adverse effects of accumulating objects in orbit (risks of collision and explosion, congestion of orbital space, emissions of pollutants during launches, fallout of objects at end-of-life, space observation), anticipating that the damage will be negligible on their individual scale. This same reason will also lead them to make poor decisions on satellite design, concerning, for example, operational life, shielding and end-of-life de-orbiting devices (Guyot and Rouillon, 2023). Finally, the absence of regulation will also compromise the emergence of an orbit cleaning sector, due to the absence of a solvent demand (Guyot and Rouillon, 2023).

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A few articles go beyond this simplified analytical framework, in order to better formalize the nature and conditions of competition in the satellite services market.

Adilov, Alexander and Cunningham (2015), and Guyot, Rao and Rouillon (2023) formalize the market for satellite services using models of horizontal and/or vertical differentiation. They consider sequential games, where a duopoly first decides on the degree of differentiation of their satellite services, before engaging in price competition. More specifically, Adilov, Alexander and Cunningham (2015) model a location game à la Salop (1979), in which the firms decide the position of their satellite in geostationnary Earth orbit. Guyot, Rao and Rouillon (2023) model a vertical differentiation game, in which the firms decide several design options of their satellite constellation (altitude and size), which then condition their service quality.

Guyot and Rouillon (2023), and Bernhard, Deschamps and Zaccour (2023) formalize a competitive game à la Cournot. Guyot and Rouillon (2023) focus on the stationary state of the orbital environment, reducing themselves to a static game. Oligopolistic firms make choices about the design and launch of their satellites, and their debris removal effort. They reach the following conclusions. Insofar it is *highly* concentrated, the satellite industry may have an interest in cleaning up the orbit. However, the size of the operating satellite fleet and the loss of social surplus increases with the number of actors in the market. Bernhard, Deschamps and Zaccour (2023) solve a dynamic game, in which duopolistic firms choose their number of launches per period, in a context where the debris stock is kept constant by the exogenous action of a space agency in charge of debris removal.

This paper is the first to use a monopolistic competition model (Dixit and Stiglitz, 1977; Spence, 1977) to formalize the market for satellite services. This analytical framework endogenizes the variety of services offered by satellite operators. By construction, it implies that firms compete for the share of the overall market, but enjoy a monopoly position on the specific market for their variety. At market equilibrium, the degree of market segmentation depends on consumers' preferences for diversity. If they value diversity, consumers will be offered a wide range of satellite services, but the quantity offered of each satellite service will be rationed, and vice versa. This trade-off will also be influenced by the initial cost of developing a new satellite service and by the unit cost of satellites.

Compared with the baseline model of monopolistic competition, the originality of our application to the satellite services market lies in the fact that firms also compete to share orbital space. The finite volume of the Earth's orbit limits its capacity to accommodate artificial satellites. As orbital space becomes saturated with objects, the operational life of satellites shortens, due to the avoidance maneuvers they have to perform to avoid collisions and/or due to collisions that could not be avoided. As satellite operators are forced to renew their satellite fleets more rapidly, the cost of maintaining service quality rises with orbital space congestion. Formally, this means that this paper also contribute to the industrial organization literature by solving a monopolistic competition model in which the unit cost of firms increases with market size, due to congestion. This could have other applications.

This paper is organized as follows. Section 2 describes our model. Section 3 solves and analyses the monopolistic equilibrium. Section 4 derives the social optimum. Section 5 compares the two outcomes. In Section 6, we give a numerical application. Section 7 concludes. Supplementary materials and proofs are supplied in an appendix.

2 The Model

Consider the satellite services market. Firms initially spend f to enter the market, representing an initial R&D investment. Active firms are indexed by $i \in [0, n]$, each supplying a different service, such that n stand for both the number active satellite operators and the diversity of satellite services.[1] The cost of building and launching a satellite is c. Upon entry, each firm i chooses its supply of satellite

[1] For the sake of simplicity, n can be any positive real number.

service, by putting into orbit a fleet of $z(i)$ satellites. Next, by assumption, it commits to replace every dead satellites instantaneously, i.e. $(\lambda + \mu) z(i)$ per period, where λ is their rate of failure and μ their rate of collision. Firm i's total (annualized) cost, including replacement of dead satellites and amortization of initial investment, is therefore equal to $(\lambda + \mu) cz(i) + \delta (cz(i) + f)$, where δ is the discount rate. The revenue flow associated with a fleet of $z(i)$ satellites is $p(i) z(i)$, where $p(i)$ is the annual rent per satellite. Below, the total number of active satellites will be denoted $z = \int_0^n z(i) \, \mathrm{d}i$.

The collision rate μ is assumed to verify:

Assumption 1. There exists $k > 0$ such that $\mu : z/k \in [0, 1) \to \mu(z/k) \in \mathbb{R}^+$ satisfies: $\mu(0) = 0$, $\mu'(z/k) \geq 0$, $\mu''(z/k) \geq 0$ and $\lim_{z/k \to 1} \mu(z/k) = \infty$.

Under assumption 1, parameter k is interpreted as the orbital carrying capacity, i.e., the maximum number of *active* satellites that the orbit can sustainably accommodate. The rate of collision μ is increasing and convex with the degree of congestion z/k. When the orbit becomes saturated with satellites (i.e., z/k tends towards 1), any satellite in orbit is quickly destroyed and must be replaced.

The preferences of a representative consumer are given by a quasi-linear utility function:

$$U(C, Z) = C + b \ln(S),$$

where C and S denote respectively the consumption of a numéraire good and the consumption of a composite of satellite services. The parameter b determines the propensity to pay for satellite services.[2] The aggregate S is represented by a constant elasticity of substitution function:

$$S = \left(\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i \right)^{1/\sigma},$$

where $\sigma \in (0, 1)$ and $\omega(i) > 0$, for all i. The parameter σ determines the consumers' preference for satellite service diversity.[3] The weights $\omega(i)$ allow heterogeneous preferences for varieties of satellite services to be formalized.

The coefficients $\omega(i)$ are assumed to satisfy:

Assumption 2. There exists $h : i \in \mathbb{R}^+ \to h(i) \in \mathbb{R}^+$, satisfying $h(i) > 0$, $h'(i) \leq 0$ and $h''(i) \geq 0$, for all i, such that $\omega(i) = nh(i) / \int_0^n h(i) \, \mathrm{d}i$.

3 Monopolistic equilibrium

In this section, we calculate the monopolistic equilibrium and derive its properties. A monopolistic equilibrium arises when each firm maximizes its profit and entry occurs until the marginal firm just breaks even (Dixit and Stiglitz, 1977; Spence, 1977).

Given prices $p(i)$ for all $i \in [0, n]$, the consumers choose C and $z(i)$ for all $i \in [0, n]$ to maximize:

$$C + \frac{b}{\sigma} \ln \left(\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i \right),$$

subject to the budget constraint:

$$C + \int_0^n p(i) z(i) \, \mathrm{d}i = I + \Pi,$$

where $I + \Pi$ represents the income in terms of the numéraire, including the initial endowment and profits distributed to the consumers.

[2] In equilibrium, the consumer's total expenditure on satellite services is equal to b.

[3] The elasticity of substitution between two varieties is equal to $1/(\sigma - 1)$.

The lagrangian of this problem writes:

$$L = C + \frac{b}{\sigma} \ln \left(\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i \right) - \nu \left(C + \int_0^n p(i) z(i) \, \mathrm{d}i - I - \Pi \right),$$

where ν is the multiplier associated to the budget constraint.

The consumers' equilibrium choice satisfies the necessary conditions:

$$\frac{\partial L}{\partial C} = 1 - \nu = 0$$

and:

$$\frac{\partial L}{\partial z(i)} = b \frac{\omega(i)^\sigma z(i)^{\sigma-1}}{\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i} - \nu p(i) = 0,$$

for all $i \in [0, n]$.

From this, we can directly derive variety i's (inverse) demand function:

$$p(i) = b \frac{\omega(i)^\sigma z(i)^{\sigma-1}}{\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i}. \tag{1}$$

Note that this implies that the total expenditure on the satellite services market, $\int_0^n p(i) z(i) \, \mathrm{d}i$, is equal to b. Below, we shall thus interpret parameter b as the "size" of the satellite services market.

Firm i's profit is equal to:

$$\pi(i) = p(i) z(i) - \left(\delta + \lambda + \mu \left(\frac{1}{k} \int_0^n z(i) \, \mathrm{d}i \right) \right) c z(i) - \delta f,$$

where, from the consumers' equilibrium, the price for satellite service supplied by firm i is given by (1).

By assumption, firm i spends f to enter the market whenever it can expect a positive profit from its satellite service, in which case it chooses $z(i)$ to maximize its profit. Under the assumption of atomicity[4], firm i's equilibrium choice satisfies the necessary conditions:

$$\frac{\partial \pi(i)}{\partial z(i)} = \sigma b \frac{\omega(i)^\sigma z(i)^{\sigma-1}}{\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i} - \left(\delta + \lambda + \mu \left(\frac{1}{k} \int_0^n z(i) \, \mathrm{d}i \right) \right) c = 0, \tag{2}$$

for all $i \in [0, n]$, and

$$\pi(n) = b \frac{\omega(n)^\sigma z(n)^\sigma}{\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i} - \left(\delta + \lambda + \mu \left(\frac{1}{k} \int_0^n z(i) \, \mathrm{d}i \right) \right) c z(n) - \delta f = 0. \tag{3}$$

The first condition means that firm i behaves as a monopolist in its market, equalizing the marginal revenue and marginal cost of an additional satellite. The second condition implies that the marginal firm, offering variety n, makes no profit. Its revenue $p(n) z(n)$ therefore just covers its variable cost $(\delta + \lambda + \mu) c z(n)$ and its fixed cost δf. Equivalently, its average revenue $p(n)$ is equal to its average cost $(\delta + \lambda + \mu) c + \delta f / z(n)$. Given that $\omega(i)$ is non-increasing, all firms $i \leq n$ make a non-negative profit and thus enter the market.

From this, we can derive:

[4] Firm i takes both $\int_0^n (\omega(i) z(i))^\sigma \, \mathrm{d}i$ and $\int_0^n z(i) \, \mathrm{d}i$ as given.

Proposition 1*: (*Characterization*) The monopolistic equilibrium, defined by $z^*(i)$, for all $i \in [0, n^*]$, z^* and n^*, satisfies:

$$z^*(i) = \frac{\omega(i)^{\frac{\sigma}{1-\sigma}}}{\int_0^{n^*} \omega(i)^{\frac{\sigma}{1-\sigma}}\,\mathrm{d}i} z^*,$$

$$z^* = \frac{\sigma}{\delta + \lambda + \mu(z^*/k)} \frac{b}{c}$$

and:

$$\int_0^{n^*} \left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i = (1-\sigma)\frac{b}{\delta f}.$$

The first equation in proposition 1* shows that the aggregated satellite fleet z^* is allocated among the operators depending on the consumers' preference parameters σ and $\omega(i)$. Under assumption 2, $z^*(i)$ is (weakly) decreasing with i.[5] The second equation in proposition 1* implicitly defines z^*, the size of the overall satellite fleet, as a function of parameters b, c, k, δ, λ and σ. The last equation implicitly defines n^*, the number of satellite service varieties in monopolistic equilibrium, as a function of parameters b, f, δ and σ.

It can be shown that the monopolistic equilibrium exists and is unique. The proof, relegated to the appendix, follows from the continuity and monotonicity of the conditions in Proposition 1*.

Using the implicit function theorem on proposition 1*, we show in appendix:

Proposition 2*: (*Comparative statics*) Under monopolistic competition:
(i) the number of in-orbit satellites z^* is increasing in b/c, k and σ, and decreasing in $\delta + \lambda$;
(ii) the equilibrium diversity of satellite services n^* is increasing in $b/(\delta f)$ and decreasing in σ.

Result (i) shows that the number of satellites in orbit increases with the size of the market for satellite services and the carrying capacity of the orbit, and decreases with the cost of a satellite, the consumers' preference for diversity, the discount rate and the failure rate. Most results are intuitive. However, the role of consumers' preference for diversity in satellite services deserves to be explained in more detail.

A decrease in parameter σ (i.e., a larger preference for diversity) has two effects. On the one hand, it results in less elastic demand for the satellite services, giving operators more monopoly power. They will thus increase their prices by reducing the size of their satellite fleet. In the short run, this leads to a decrease in the total number of satellites owned by existing operators. On the other hand, a smaller parameter σ also attracts new operators to enter the market, due to the increased profitability of each satellite service. In the long run, this implies more satellites in orbit as newcomers deploy their own fleets. The combination of the two effects result in an ambiguous outcome. Our results show a reduction in the total number of satellites in orbit, indicating that the short-term effect prevails.

Result (ii) shows that the diversity of satellite services increases with the size of the market and the consumers' taste for diversity, and decreases with the discount factor and the cost of the initial R&D investment. All these results are intuitive. On the other hand, the diversity of satellite services does not depend on the carrying capacity of orbital space.

[5] $z^*(i)$ is constant if $h'(i) = 0$.

4 Social optimum

In this section, we calculate the social optimum and derive its properties. The first best optimum is defined as a situation that maximizes the consumers' utility given resource endowment and available technologies.

The social problem is to choose $z(i)$ for all $i \in [0, n]$ and n to maximize:

$$C + \frac{b}{\sigma} \ln \left(\int_0^n \left(\omega(i) z(i) \right)^\sigma \mathrm{d}i \right),$$

subject to the (intertemporal) budget constraint:

$$C + \left(\delta + \lambda + \mu \left(\frac{1}{k} \int_0^n z(i) \, \mathrm{d}i \right) \right) c \int_0^n z(i) \, \mathrm{d}i + \delta f n = I.$$

Given that $z(i)$ and $z(j)$, for all i and j, contribute equally in the budget constraint, by convexity, an optimal solution should equate their marginal rate of substitution, i.e. $\omega(i)^\sigma z(i)^{\sigma-1} = \omega(j)^\sigma z(j)^{\sigma-1}$ for all i and j. This implies that:

$$\int_0^n \left(\omega(i) z(i) \right)^\sigma \mathrm{d}i = \left(\int_0^n \omega(i) \, \mathrm{d}i \right)^{1-\sigma} z^\sigma.$$

Thus the social problem is equivalent to choose z and n to maximize:

$$C + \frac{1-\sigma}{\sigma} b \ln \left(\int_0^n \omega(i)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i \right) + b \ln(z),$$

subject to:

$$C + \left(\delta + \lambda + \mu \left(\frac{z}{k} \right) \right) cz + \delta f n = I.$$

The lagrangian of this problem writes:

$$L = C + \frac{1-\sigma}{\sigma} b \ln \left(\int_0^n \omega(i)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i \right) + b \ln(z) - \nu \left(C + \left(\delta + \lambda + \mu \left(\frac{z}{k} \right) \right) cz + \delta f n - I \right)$$

where ν is the multiplier associated to the budget constraint.

The social optimum satisfies the necessary conditions:

$$\frac{\partial L}{\partial C} = 1 - \nu = 0, \tag{4}$$

$$\frac{\partial L}{\partial z} = \frac{b}{z} - \nu \left(\left(\delta + \lambda + \mu \left(\frac{z}{k} \right) \right) c + \mu' \left(\frac{z}{k} \right) c \frac{z}{k} \right) = 0 \tag{5}$$

and:

$$\frac{\partial L}{\partial n} = \frac{1-\sigma}{\sigma} b \frac{\omega(n)^{\frac{\sigma}{1-\sigma}}}{\int_0^n \omega(i)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i} - \nu \delta f = 0. \tag{6}$$

Condition (5) means that a social optimum equates the marginal utility and marginal social cost of an additional satellite. Comparing it with condition (2), which applies in the case of monopolistic equilibrium, we observe two differences. Firstly, the marginal utility of an additional satellite, given by b/z, is greater than the marginal revenue of an additional satellite, given by $\sigma b/z$. This means that firms in monopolistic equilibrium will tend to place less satellites in orbit. Secondly, because it includes the external cost $\mu'(z/k)\,cz$, the marginal social cost of an additional satellite is greater than the marginal private cost. Since this externality is ignored in monopolistic equilibrium, firms will tend to place more satellites in orbit. All in all, these two opposing effects imply that comparing the sizes of the total in-orbit satellite fleet in the monopolistic equilibrium and in the social optimum will be ambiguous.

Condition (6) equates the utility of a marginal variety n with the cost of entry. Given that $\omega(i)$ is non-increasing, all inframarginal varieties $i \leq n$ improve social welfare. Comparing this with condition (3) in the case of monopolistic equilibrium determination, we see that firms recover only a fraction of the surplus generated by an additional variety. It is therefore in their interest to limit the number of varieties of satellite services offered.

The following proposition reformulates the previous conditions to characterize an optimal state:

Proposition 1°. (*Characterization*) The social optimum, defined by $z^\circ(i)$, for all $i \in [0, n^\circ]$, z° and n°, satisfies the necessary conditions:

$$z^\circ(i) = \frac{\omega(i)^{\frac{\sigma}{1-\sigma}}}{\int_0^{n^*} \omega(i)^{\frac{\sigma}{1-\sigma}}\,\mathrm{d}i} z^\circ,$$

$$z^\circ = \frac{1}{\delta + \lambda + \mu\left(\frac{z^\circ}{k}\right) + \mu'\left(\frac{z^\circ}{k}\right)\frac{z^\circ}{k}} \frac{b}{c}$$

and

$$\int_0^{n^\circ} \left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i = \frac{1-\sigma}{\sigma}\frac{b}{\delta f}.$$

The first equation in proposition 1° shows that the aggregated satellite fleet z° is allocated among the operators depending on the consumers' preference parameters σ and $\omega(i)$. The second equation implicitly defines z°, the size of the overall satellite fleet at the social optimum, as a function of parameters b, c, k, δ and λ. Note that it does not depend on σ. The last equation implicitly defines n°, the number of satellite service varieties at the social optimum, as a function of parameters b, f, δ and σ.

It can be shown that the social optimum exists and is unique. The proof, relegated to the appendix, follows from the continuity and monotonicity of the conditions in Proposition 1°.

Using the implicit function theorem on proposition 1°, we show in appendix:

Proposition 2°. (*Comparative statics*) At a social optimum:
(i) the number of in-orbit satellites z° is increasing in b/c and k, and decreasing in $\delta + \lambda$;
(ii) the diversity of satellite services n° is increasing in $b/(\delta f)$ and decreasing in σ.

Result (i) shows that the number of satellites in orbit increases with the size of the market and the orbit carrying capacity, and decreases with the cost of a satellite, the discount rate and the failure rate. All these results are intuitive. It should be noted, however, that the number of satellites in orbit does not depend on consumers' preference for diversity. In connection with result (ii), this means that, on average, the arrival of a new variety of service takes place at constant overall satellite fleet, i.e. by reducing the size of existing fleets.

Result (ii) shows that the diversity of satellite services increases with the size of the market and the consumers' preference for diversity, and decreases with the discount factor and the cost of the initial R&D investment. On the other hand, it does not depend on the carrying capacity of orbital space.

5 Comparison

We compare here the monopolistic equilibrium with the social optimum, looking at both the degree of congestion in the orbit and the diversity of services offered.

In our setting, there are two reasons why the monopolistic equilibrium may deviate from the social optimum: monopoly power and collision risk. As we saw above, these two forces are working in opposite directions. The comparison will thus be ambiguous, with the satellite industry either overusing or underusing Earth orbit, as the case may be. Proposition 3 below confirms this, but sets a condition that removes the ambiguity.

In the statement of proposition 3, we will use the following assumption:

Assumption 3. The elasticity of the risk of collision $\phi(\rho) = \mu'(\rho)\rho/\mu(\rho)$ is non decreasing (i.e., $\phi'(\rho) \geq 0$) for all $\rho \in [0,1)$.

We show the following:

Proposition 3. (*Comparison*) Monopolistic equilibrium *vs* social optimum.
(i) There exists $g : k \in (0,\infty) \to g(k) \in (0,1)$ such that $z^* \gtreqless z^\circ$ if and only if $\sigma \gtreqless g(k)$. Moreover, under assumption 3, $\lim_{k\to 0} g(k) = 0$, $\lim_{k\to\infty} g(k) = 1$ and $g'(k) \geq 0$ for all k;
(ii) $n^* < n^\circ$ for all σ.

The proof is given in the appendix. The proof of part (i) proceeds as follows. We first derive the set of parameters k and σ such that $z^* = z^\circ = z$ for some $z \in [0,k)$. We then show that this set implicitly defines a frontier $\sigma = g(k)$ in the (O,k,σ) plane. We finally note that z^* increases with σ, whereas z° does not depend on σ, implying that $z^* > z^\circ$ (respectively, $z^* < z^\circ$) above (below) this frontier. The proof of part (ii) is immediate.

Proposition 3 shows that overexploitation of the Earth's orbit occurs when the carrying capacity of the Earth's orbit is reduced and/or when consumers show a low preference for the diversity of satellite services. Here we identify the two causes of sub-optimal use of the Earth's orbit by the satellite industry. The first is firms' market power, which will determine the scale of satellite fleets deployed. A low preference for diversity, synonymous with low market power, justifies the deployment of large satellite fleets. The second relates to the scale of the externality associated with space activity, which is ignored by satellite operators. The space industry will thus deploy an oversized satellite fleet in the event of limited carrying capacity, in which case the risk of collision grows rapidly with the number of satellites in orbit.

6 Numerical application

We specify and calibrate here our model to match real data. We perform a sensitivity analysis with respect to the orbital carrying capacity and the consumers' preference for service diversity.

The specification of the risk of collision μ is based on the physical model described in appendix A1:

$$\mu\left(\frac{z}{k}\right) = \lambda\frac{z}{k-z}.$$

The specification of the weights $\omega(i)$ is given by:[6]

$$\omega(i) = \frac{\gamma n}{1-e^{-\gamma n}}e^{-\gamma i}.$$

Our calibration of the economic parameters relies on the following data. Total revenues for satellite manufacturers and launchers ranged from \$10,800 million to \$25,700 million over the period 2001 to

[6] In line with assumption 2, this implies that $h(i) = \gamma e^{-\gamma i}$.

2023 (SIA, 2002 to 2024). Over the same period, the number of satellites launched ranged from 80 to 2,844 satellites per year (Celestrak, 2024). According to our calculation, this gives an estimate of the unit cost of satellites, of an order of \$2.3 million/sat. for low Earth orbits and of \$361.5 million/sat for higher altitudes.[7] Besides, the total revenue from satellite services ranged from \$110,200 million to \$126,500 million over the period 2018 to 2023 (SIA, 2019 to 2024). A typical satellite in low Earth orbit has a planned lifetime of 4.4 years (UCS, 2023). Finally, we use a discount rate of 5 % a year.

Our benchmark calibration is summarized in Table 1.[8] Parameters k and σ will be subjected to a sensitivity analysis. Our results are displayed in Table 2 and Figure 2.

Coefficient	Estimate	Unit	Sources
b	$118,000$	\$m	SIA (2019 to 2024)
c	3.6	\$m/sat.	Celestrak (2024) and SIA (2001-2024)
f	360	\$m	Conjectured
δ	5	%/y.	Conjectured
γ	0.75	/	Conjectured
$1/\lambda$	4.4	y.	UCS (2023)
k	$\left(0, 10^5\right]$	sat.	Sensitivity analysis
σ	$(0, 1)$	/	Sensitivity analysis

Table 1: Benchmark Calibration

Consider first the benchmark simulation such that $k = 25,000$ sat. and $\sigma = 0.5$.

Table 2 summarizes the most relevant outcomes. At the monopolistic equilibrium, the satellite operators maintain a fleet of $z^* = 18,286$ satellites and offer a diversity of $n^* = 10$ satellite services. They launch $q^* = 15,475$ satellites every year to replace dead satellites. The probability of a satellite being destroyed during its mission is $P^* = 73$ %, so its operational lifetime is reduced to $1/(\lambda^* + \mu^*) = 1.18$ years. In the optimal policy, the satellite operators maintain a fleet of $z^o = 16,449$ satellites and offer a diversity of $n^o = 11$ satellite services. They launch $q^o = 10,930$ satellites every year to replace dead satellites. The probability of a satellite being destroyed during its mission is $P^o = 66$ %, so its operational lifetime is reduced to $1/(\lambda^o + \mu^o) = 1.5$ years.

	Fleet (sat.)	Service diversity (serv.)	Launch (sat./y.)	Risk of collision (%)	Actual lifetime (years)
Monop. comp.	$18,286$	10	$15,475$	73	1.18
Optimum	$16,449$	11	$10,930$	66	1.5

Table 2: Equilibrium and optimal outcomes

We finally propose a sensitivity analysis, varying parameter k between 0 and 100,000 satellites and parameter σ between 0 and 1. Our results are presented in Figure 1. The baseline simulation ($k = 25,000$ sat. and $\sigma = 0.5$) is recalled by the red points in the plots.

[7] In accordance with our physical model, we normalized the units by considering satellites of mass equal to 500 kg. For the record, in 2023, the average mass of a satellite was 317 kg for low Earth orbit, against 3398 kg for higher altitudes (UCS, 2023).

[8] The data collected and calibration steps are explained in a supplementary document.

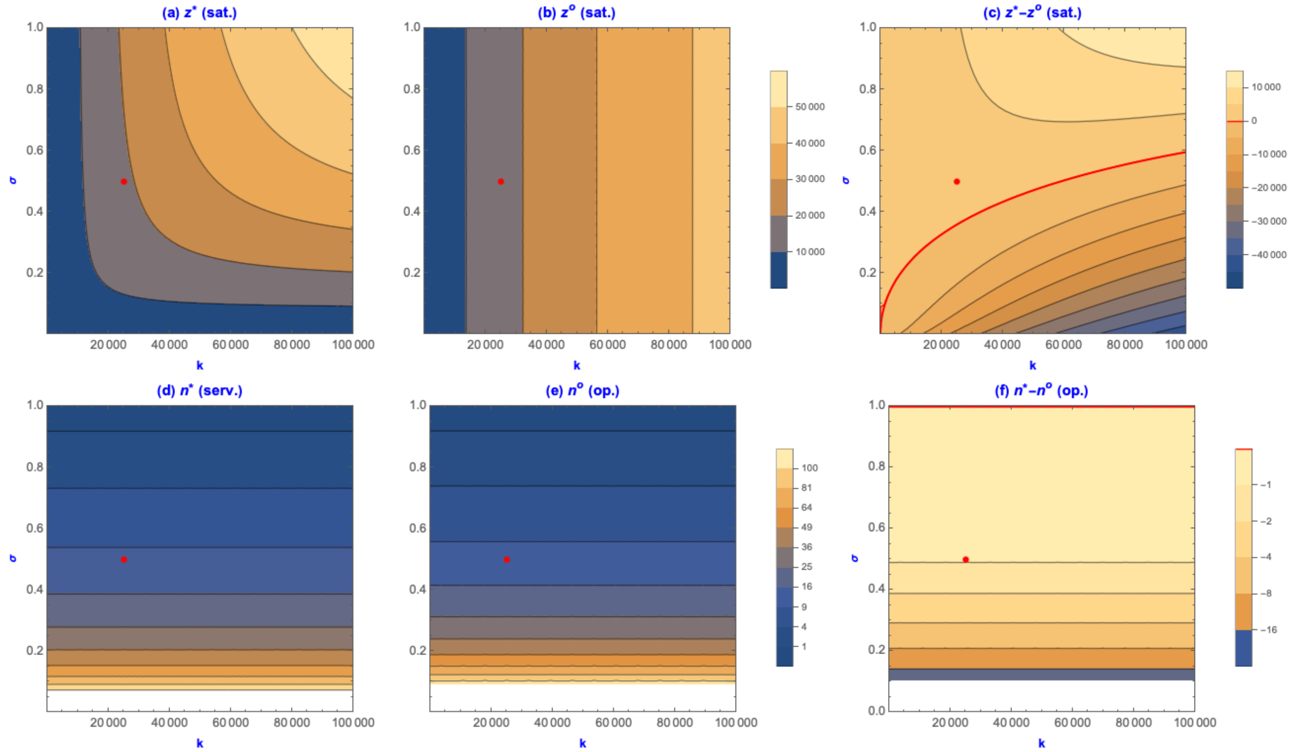

Figure 1: Sensitivity analysis ($0 < k \leq 10^5$ and $0 < \sigma < 1$)

Let's start by considering the effect of the orbit's carrying capacity k. This parameter formalizes the environmental constraint of the Earth's orbit. It depends on both physical constants (the volume of the orbit, the speed and lifetime of objects in orbit, etc.) and technological choices (the mass and size of satellites, their ability to detect and maneuver to avoid collisions, mitigation measures, etc.). Formally, the smaller the parameter k, the greater the risk of collision for a given population of active satellites in orbit. Panels (a) and (b) in Figure 1 show that the size of the satellite fleet in orbit increases with k, both in monopolistic equilibrium (z^* in panel (a)) and in the social optimum (z° in panel (b)). Greater orbital carrying capacity means less risk of collision, all else being equal. This reduces satellite mortality due to collision and thus lowers replacement costs. In the limit, when $k \to \infty$, the environmental constraint is no longer effective, so that only economic conditions (i.e., consumers' preference and operators' technology) determine the satellite fleet. Another way to measure the degree of orbit exploitation is to calculate the congestion rates (z^*/k and z°/k). Our results show that the population of satellites in orbits grows less than proportionally with k, so that congestion rates decrease with k. We can also show that, both in the monopolistic equilibrium and in the social optimum, the congestion rates tend towards 1 when $k \to 0$ and towards 0 when $k \to \infty$. We can see from panels (d) and (e) in Figure 1 that the diversity of satellite services does not vary with k, neither in the monopolistic equilibrium (n^* in panel (d)) nor in the social optimum (n° in panel (e)). Given that the number of active satellites in orbit increases with k, this means that a larger orbital carrying capacity is used to improve the supply of existing satellite services, not to develop and market new services. As a corollary, the fleet size of each incumbent operators increases with k. Let us now consider the effect of the preference parameter σ. It is an indicator of the consumers' preference for satellite services diversity. When $\sigma = 1$, consumers see satellite services as perfectly substitutable. They thus have the same propensity to pay for one satellite fleet, offering a single service, or several satellite fleets, with the same total number of satellites, offering differentiated services. When $\sigma < 1$, consumers show a preference for diversity. They are willing to pay more for the option with several fleets providing different services.

It can be seen from panels (a) and (b) in Figure 1 that z^* increases with σ, while z° does not vary with σ. Explanations for this are given below, distinguishing between a short-period effect (i.e., changes in the choices made by incumbent operators) and a long-period effect (i.e., arrival of new operators or exit of incumbent operators).

A reduction in the parameter σ means a decline in the elasticity of demand for each satellite service, thereby reinforcing the monopoly power of operators within their respective markets. Consequently, they will exploit this to increase prices by reducing the size of their satellite fleet. In the short term, the total number of satellites owned by existing operators will therefore decrease.

On the other hand, panel (d) in Figure 1 shows that, in a monopolistic equilibrium, a reduction in the parameter σ results in a greater number of satellite services. This is due to the fact that new market entrants are attracted by the enhanced profitability of each satellite service. Consequently, the deployment of newcomers' satellite fleets will lead to an increase in the number of satellites in orbit.

Overall, the two effects, in opposite directions, yield an ambiguous result. Our results, showing a net reduction in the total number of satellites in orbit, prove that the first effect prevails. Thus, a smaller parameter σ induces a reorganization of the satellite services market, with more services and each operator owing a smaller fleet of satellites.

In the case of the social optimum, as the parameter σ decreases, consumers' greater preference for diversity also drives the development of more satellite services (see panel (e) in Figure 1). However, this occurs at a constant size of the total satellite fleet (see panel (b) in Figure 1). In other words, the same number of satellites is simply divided among more players and services.

Panels (c) and (f) in Figure 1 are used to compare the monopolistic equilibrium and the social optimum.

The graph in panel (c) of Figure 1 plots the difference between the total number of satellites in orbit at the monopolistic equilibrium and at the social optimum (i.e., $z^* - z^\circ$). The red boundary depicts the location of points where the two fleets have same size. Above it, the monopolistic equilibrium induces an overexploitation of the orbit (i.e., $z^* > z^\circ$), relative to the social optimum, and vice versa. This can be explained as follows.

In a monopolistic equilibrium, satellite operators, by assumption, neglect their contribution to orbit congestion and collisions. Their choices therefore depend solely on the competitive considerations regarding the satellite services market. As the parameter σ decreases, the satellite services market is segmented into an increasing number of operators offering differentiated services. Each operator then enjoys an increasing monopoly power in an ever-narrower niche market. For sufficiently small value of σ, monopoly power becomes so strong that it becomes in the operators' best interest to drastically ration their market, resulting in an under-exploitation of the Earth orbit compared with the optimal state.

Added to the above mechanism is the fact that the monopolistic market offers an insufficient number of satellite services, as shown in panel (f) of Figure 1 (i.e., $n^* - n^\circ < 0$). The explanation for this is that monopolistic competition produces imperfect price discrimination, so that operators are unable to fully capture the consumers' willingness to pay for satellite services. Consequently, they are under-incentivized to invest in R&D for new satellite services. Finally, there are fewer satellite operators and fewer satellite fleets at the monopolistic equilibrium than at the social optimum.

7 Conclusion

In this paper, we have applied a monopolistic competition model to analyze the satellite services sector. Unlike the existing literature, this framework endogenizes the diversity of satellite service offerings and the associated market dynamics. Firms enter the market to offer new services and deploy dedicated satellite fleets. We have crossed this issue with that of orbital pollution by space debris. In our model, space congestion depends on the number of services offered and the size of the satellite fleets deployed for each service. The associated risk of collision therefore constrains both the diversity and quantity of services offered.

We show that the equilibrium in monopolistic competition is sub-optimal, as firms manipulate prices and neglect the externalities associated with their activity. We show that the number of satellites deployed in orbit will be too large if Earth orbit carrying capacity and/or consumer preferences for diversified services are low. On the other hand, in all cases, the range of services offered will be too limited. These results are demonstrated analytically and illustrated by a numerical application calibrated on real data.

8 Appendix

A.1 Physical model

We describe here a physical model of low Earth orbit, based on Farinella and Cordelli (1991) and Lafleur (2011). They introduce a system of two coupled differential equations, one for the population of big objects and the other for the population of debris fragments. Big objects include intact satellites, *either active or inactive*, and rocket bodies. They typically have a cross-section of a few square meters and a mass of hundreds of kilograms. Debris fragments are objects generated by collisions or explosions, with a size of a few centimeters and a mass of a few grams, capable of causing catastrophic breakup when impacting a satellite.

The main difference of our model with Farinella and Cordelli (1991) and Lafleur (2011) is that we divide the set of big objects into two parts, in order to distinguish active satellites from other inactive big bodies. This implies introducing a third differential equation for the population of operational satellites. Unless assuming infinitely lived satellites, as Rao and Rondina (2019) do, this extension is necessary to formalize the operational lifetime.

We use the following notations:

$$q(t) = \text{rate of active satellites launched;}$$

$$x(t) = \text{population of debris fragments;}$$

$$y(t) = \text{population of inactive satellites;}$$

$$z(t) = \text{population of active satellites.}$$

We formalize the evolution of the orbital environment with the following dynamical system:[9]

[9] To simplify, we neglect the orbit decay of intact satellites, and collisions between debris fragments (i.e., x-x) and between intact satellites (i.e., y-y, y-z and z-z). The same simplifying assumptions are also used by Farinella and Cordelli (1991) and Lafleur (2011). They are justified as follows. Orbit decay of intact satellites is found to be negligible by Lafleur (2011). Collisions between debris fragments generate second-order fragments too small to cause catastrophic breakup. Collisions between intact satellites is negligible according to simulations by Lafleur (2011). This is even more true for operational satellites, which can perform avoidance manoeuvres, knowing that intact satellites are large enough to be traced by radar.

$$\begin{cases} \dot{x}(t) = \underline{\alpha} q(t) - \beta x(t) + \eta \theta x(t)(y(t) + z(t)), & x(0) = x_0, \\ \dot{y}(t) = \overline{\alpha} q(t) + \lambda z(t) - \theta x(t) y(t), & y(0) = y_0, \\ \dot{z}(t) = q(t) - (\lambda + \theta x(t)) z(t), & z(0) = z_0. \end{cases}$$

The first differential equation describes the evolution of the population of debris fragments, $x(t)$. The first term, $\underline{\alpha} q(t)$, are fragments released as a byproduct of satellite launches (i.e., explosion of rocket bodies and space objects), with $\underline{\alpha} > 0$ the number of fragments per launch. The second term, $\beta x(t)$, represents the decay of the stock of debris fragments due to the atmospheric drag, with $\beta > 0$ the inverse of their average orbital lifetime. The fourth term, $\eta \theta x(t)(y(t) + z(t))$, gives the addition of debris fragments generated by collisions of debris fragments with intact satellites (either active or inactive), with $\eta > 0$ the numbers of fragments per collision and $\theta x(t) \geq 0$ the rate of collision per unit of intact satellite.

The second differential equation represents the evolution of the population of inactive satellites, $y(t)$. The first component, $\overline{\alpha} q(t)$, refers to the rate of inactive satellites released by launching activities (i.e., rocket upper stages), with $\overline{\alpha} > 0$ their number per launch. The third term, $\lambda z(t)$, represents operational satellites arriving at the end of their lifetime, thus becoming inactive, with $\lambda > 0$ the inverse of their average operational lifetime. The last term, $\theta x(t) y(t)$, refers to the number of non-operational satellites destroyed as a result of collisions.

The last differential equation gives the evolution of the population of operational satellites, $z(t)$. The first component, $q(t)$, is the result of lauching activity by the space sector. The last term, $(\lambda + \theta x(t)) y(t)$, refers to the number of satellites that cease to operate, either for technical reasons (i.e., fuel, breakdowns) or environmental reasons (i.e., collisions).

Now, let us consider a constant fleet of active satellites forever, i.e., $z(t) = z$ for all t. This necessitates that satellite operators replace immediately every defunct satellites:

$$q(t) = (\lambda + \theta x(t)) z$$

for all t. Upon substitution, the dynamical system then simplifies as:

$$\begin{cases} \dot{x}(t) = \underline{\alpha} \lambda z + ((\underline{\alpha} + \eta) \theta z - \beta) x(t) + \eta \theta x(t) y(t), & x(0) = x_0, \\ \dot{y}(t) = (1 + \overline{\alpha}) \lambda z + \overline{\alpha} \theta z x(t) - \theta x(t) y(t), & y(0) = y_0. \end{cases}$$

We wish to calculate the stationary states of the dynamical system, i.e., $x(t) = x^*$ and $y(t) = y^*$ such that $\dot{x}(t) = \dot{y}(t) = 0$ for all t. It is immediate to calculate from the system:

$$\dot{x}(t) = \underline{\alpha} \lambda z + ((\underline{\alpha} + \eta) \theta z - \beta) x^* + \eta \theta x^* y^* = 0$$

$$\dot{y}(t) = (1 + \overline{\alpha}) \lambda z + \overline{\alpha} \theta z x^* - \theta x^* y^* = 0$$

that:

$$x^* = \frac{\lambda}{\theta} \frac{z}{k - z},$$

$$y^* = (1 + \overline{\alpha}) k - z,$$

where:

$$k = \frac{1}{\underline{\alpha} + (1 + \overline{\alpha}) \eta} \frac{\beta}{\theta}.$$

Assuming that $z < k$, all results are strictly positive. This condition is physically necessary in the long run.

The above closed-form expressions rationalize our model in Section 2. Indeed, we have postulated the existence of a parameter k and a function $\mu(z/k)$, giving the long-term collision risk as a function of the congestion rate. Our calculus here suggests that a possible specification is:

$$\mu\left(\frac{z}{k}\right) = \theta x^* = \lambda \frac{z}{k-z},$$

which will be used in Section 6.

Also, let us note that the above stationary solution highlights some assumptions left implicit in the main text. Parameter k depends not only on natural parameters:

- the rate of decay of debris fragments, β;
- the rate of collision per debris and satellite, θ;[10]

but also on technological parameters under the control of the space sector:

- the number and types of objects released per launch, $\underline{\alpha}$ and $\overline{\alpha}$;
- the number of debris fragments generated by a destroyed satellite, η.

We can supply conditions such that the stationary solution is locally stable. Linearization of the dynamical system around it gives:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = A \begin{bmatrix} x(t) - x^* \\ y(t) - y^* \end{bmatrix},$$

where

$$A = \begin{bmatrix} -\beta + (\underline{\alpha} + \eta)\theta z + \eta\theta y^* & \eta\theta x^* \\ \overline{\alpha}\theta z - \theta y^* & -\theta x^* \end{bmatrix}.$$

Substitute:

$$x^* = \frac{\lambda}{\theta}\frac{z}{k-z},$$
$$y^* = (1+\overline{\alpha})k - z,$$

to show:

$$A = \begin{bmatrix} -\beta + (1+\overline{\alpha})\eta\theta k + \underline{\alpha}\theta z & \eta\frac{\lambda z}{k-z} \\ -(1+\overline{\alpha})\theta(k-z) & -\frac{\lambda z}{k-z} \end{bmatrix}.$$

Using:

$$k = \frac{1}{\underline{\alpha} + (1+\overline{\alpha})\eta}\frac{\beta}{\theta},$$

we can simplify as follows:

$$A = \begin{bmatrix} -\underline{\alpha}\theta(k-z) & \eta\frac{\lambda z}{k-z} \\ -(1+\overline{\alpha})\theta(k-z) & -\frac{\lambda z}{k-z} \end{bmatrix}.$$

A necessary and sufficient condition for this linearized dynamical system to be stable is for matrix A to have all its eigenvalues with a negative real part. The corresponding characteristic polynomial is:[11]

[10] Strictly speaking, the parameter θ depends on both physical parameters (e.g. orbit volume and objects speed) and technological parameters (e.g. satellite size).

[11] To avoid any confusion, recall that the notation λ refers in our model to the rate of technical failure of an operational satellite. This is the reason why we denote the eigenvalues by ν, departing from the usual notation.

$$\det\left(A-\nu I\right)=\nu^{2}+\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)\nu+\beta\lambda\frac{z}{k}.$$

The eigenvalues of matrix A are:[12]

$$\nu_{1}=\frac{1}{2}\left(-\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)+\sqrt{\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)^{2}-4\beta\lambda\frac{z}{k}}\right)$$

and

$$\nu_{2}=\frac{1}{2}\left(-\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)-\sqrt{\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)^{2}-4\beta\lambda\frac{z}{k}}\right).$$

As:

$$\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)>0$$

and:

$$4\beta\lambda\frac{z}{k}>0$$

for all $z<k$, there are two possible cases:

(i) If $\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)^{2}-4\beta\lambda\frac{z}{k}\geq 0$, both eigenvalues are negative real numbers, since $\sqrt{\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)^{2}-4\beta\lambda\frac{z}{k}}<\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)$;

(ii) If $\left(\frac{\lambda z}{k-z}+\underline{\alpha}\theta\left(k-z\right)\right)^{2}-4\beta\lambda\frac{z}{k}<$, both eigenvalues are imaginary numbers with negative real parts.

A.2 Existence and unicity

Monopolistic equilibrium

- Existence:

If $z=0$, then $\left(\delta+\lambda+\mu\left(z/k\right)\right)z=0<\sigma\frac{b}{c}$; if $z\to k$, then $\left(\delta+\lambda+\mu\left(z/k\right)\right)z\to\infty>\sigma\frac{b}{c}$. By continuity, there exists $0<z<k$ such that $\left(\delta+\lambda+\mu\left(z/k\right)\right)z=\sigma\frac{b}{c}$.

If $n=0$, then $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=0<\left(1-\sigma\right)\frac{b}{\delta f}$; if $n\to\infty$, then $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=\infty>\left(1-\sigma\right)\frac{b}{\delta f}$. By continuity, there exists $0<n<\infty$ such that $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=\left(1-\sigma\right)\frac{b}{\delta f}$.

- Unicity:

As $\frac{\partial}{\partial z}\left(\left(\delta+\lambda+\mu\left(z/k\right)\right)z\right)=\delta+\lambda+\mu\left(z/k\right)+\mu'\left(z/k\right)z/k>0$, the solution is unique, by monotonicity.

As $\frac{\partial}{\partial n}\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=1-\frac{\sigma}{1-\sigma}\frac{\omega'(n)}{\omega(n)}\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i>0$, the solution is unique, by monotonicity.

[12] The last two conjugate eigenvalues can be either real or complex, depending on the sign of the term under the square root.

Social optimum

- Existence:

If $z=0$, then $(\delta+\lambda+\mu(z/k)+\mu'(z/k)z/k)z=0<\frac{b}{c}$; if $z\to k$, then $(\delta+\lambda+\mu(z/k)+\mu'(z/k)z/k)z\to\infty>\frac{b}{c}$. By continuity, there exists $0<z<k$ such that $(\delta+\lambda+\mu(z/k)+\mu'(z/k)z/k)z=\frac{b}{c}$.

If $n=0$, then $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=0<\frac{1-\sigma}{\sigma}\frac{b}{\delta f}$; if $n\to\infty$, then $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=\infty>\frac{1-\sigma}{\sigma}\frac{b}{\delta f}$. By continuity, there exists $0<n<\infty$ such that $\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=\frac{1-\sigma}{\sigma}\frac{b}{\delta f}$. **- Unicity:**

As $\frac{\partial}{\partial z}(\delta+\lambda+\mu(z/k)+\mu'(z/k)z/k)z=\delta+\lambda+\mu(z/k)+3\mu'(z/k)z/k+\mu''(z/k)(z/k)^2>0$, there the solution is unique, by monotonicity.

As $\frac{\partial}{\partial n}\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=1-\frac{\sigma}{1-\sigma}\frac{\omega'(n)}{\omega(n)}\int_0^n\left(\frac{\omega(i)}{\omega(n)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i>0$, there the solution is unique, by monotonicity.

A.3 Comparative statics (proofs of Propositions 2* et 2°)

Monopolistic equilibrium

Total differentiation of the following conditions of proposition 1*:

$$\left(\delta+\lambda+\mu\left(\frac{z^*}{k}\right)\right)z^*=\sigma\frac{b}{c}$$

and:

$$\int_0^{n^*}\left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=(1-\sigma)\frac{b}{\delta f}$$

gives:

$$\left(\delta+\lambda+\mu\left(\frac{z^*}{k}\right)+\mu'\left(\frac{z^*}{k}\right)\frac{z^*}{k}\right)\mathrm{d}z^*=\begin{array}{c}\sigma\ \mathrm{d}\left(\frac{b}{c}\right)\\+\mu'\left(\frac{z^*}{k}\right)\left(\frac{z^*}{k}\right)^2\ \mathrm{d}k\\+\frac{b}{c}\ \mathrm{d}\sigma\\-z^*\ \mathrm{d}(\delta+\lambda)\end{array}$$

and:

$$\left(1-\frac{\sigma}{1-\sigma}\frac{\omega'(n^*)}{\omega(n^*)}\int_0^{n^*}\left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i\right)\mathrm{d}n^*=\begin{array}{c}(1-\sigma)\ \mathrm{d}\left(\frac{b}{\delta f}\right)\\-\left(\frac{1}{(1-\sigma)^2}\int_0^{n^*}\ln\left(\frac{\omega(i)}{\omega(n^*)}\right)\left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i+\frac{b}{\delta f}\right)\ \mathrm{d}\sigma\end{array}.$$

Note that from assumptions 1 and 2:

$$\delta+\lambda+\mu\left(\frac{z^*}{k}\right)+\mu'\left(\frac{z^*}{k}\right)\frac{z^*}{k}>0,$$

$$1-\frac{\sigma}{1-\sigma}\frac{\omega'(n^*)}{\omega(n^*)}\int_0^{n^*}\left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i>0$$

and:

$$\frac{1}{(1-\sigma)^2}\int_0^{n^*}\ln\left(\frac{\omega(i)}{\omega(n^*)}\right)\left(\frac{\omega(i)}{\omega(n^*)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i+\frac{b}{\delta f}>0.$$

Social optimum

Total differentiation of the following conditions of proposition 1°:

$$\left(\delta+\lambda+\mu\left(\frac{z^\circ}{k}\right)+\mu'\left(\frac{z^\circ}{k}\right)\frac{z^\circ}{k}\right)z^\circ=\frac{b}{c}$$

and:

$$\int_0^{n^\circ}\left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i=\frac{1-\sigma}{\sigma}\frac{b}{\delta f}$$

gives:

$$\left(\delta+\lambda+\mu\left(\frac{z^\circ}{k}\right)+3\mu'\left(\frac{z^\circ}{k}\right)\frac{z^\circ}{k}+\mu''\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^2\right)\mathrm{d}z^\circ= \begin{array}{c}\mathrm{d}\left(\frac{b}{c}\right)\\ +\left(2\mu'\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^2+\mu''\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^3\right)\mathrm{d}k\\ -z^\circ\,\mathrm{d}(\delta+\lambda)\end{array}$$

and:

$$\left(1-\frac{\sigma}{1-\sigma}\frac{\omega'(n^\circ)}{\omega(n^\circ)}\int_0^{n^\circ}\left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i\right)\mathrm{d}n^\circ= \begin{array}{c}\frac{1-\sigma}{\sigma}\,\mathrm{d}\left(\frac{b}{\delta f}\right)\\ -\left(\frac{1}{(1-\sigma)^2}\int_0^{n^\circ}\ln\left(\frac{\omega(i)}{\omega(n^\circ)}\right)\left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i+\frac{1}{\sigma^2}\frac{b}{\delta f}\right)\mathrm{d}\sigma\end{array}.$$

Note that from assumptions 1 and 2:

$$\left(\delta+\lambda+\mu\left(\frac{z^\circ}{k}\right)+3\mu'\left(\frac{z^\circ}{k}\right)\frac{z^\circ}{k}+\mu''\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^2\right)>0,$$

$$2\mu'\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^2+\mu''\left(\frac{z^\circ}{k}\right)\left(\frac{z^\circ}{k}\right)^3>0,$$

$$1-\frac{\sigma}{1-\sigma}\frac{\omega'(n^\circ)}{\omega(n^\circ)}\int_0^{n^\circ}\left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i>0.$$

and:

$$\frac{1}{(1-\sigma)^2}\int_0^{n^\circ}\ln\left(\frac{\omega(i)}{\omega(n^\circ)}\right)\left(\frac{\omega(i)}{\omega(n^\circ)}\right)^{\frac{\sigma}{1-\sigma}}\mathrm{d}i+\frac{1}{\sigma^2}\frac{b}{\delta f}>0.$$

A.4 Comparison (proof of Proposition 3)

(i) Let us compare z^* and z°, which satisfy respectively:

$$\left(\delta+\lambda+\mu\left(\frac{z^*}{k}\right)\right)z^*=\sigma\frac{b}{c}$$

and:

$$\left(\delta+\lambda+\mu\left(\frac{z^\circ}{k}\right)+\mu'\left(\frac{z^\circ}{k}\right)\frac{z^\circ}{k}\right)z^\circ=\frac{b}{c}.$$

From propositions 2^* and 2°, we know that z^* is increasing in σ, whereas z° does not depend on σ.

Consider the set of parameters k and σ such that $z^* = z^\circ = z$ for some $z \in [0, k)$. Formally, it is written as:

$$\left\{(k, \sigma) \in \mathbb{R}^+ \times (0,1)\,; \exists z \in [0,k) \text{ such that } z^* = z^\circ = z\right\}.$$

This set satisfies:

$$\left(\delta + \lambda + \mu\left(\frac{z}{k}\right)\right) z = \sigma \frac{b}{c}$$

and:

$$\left(\delta + \lambda + \mu\left(\frac{z}{k}\right) + \mu'\left(\frac{z}{k}\right)\frac{z}{k}\right) z = \frac{b}{c}.$$

Letting $\rho = z/k \in [0,1)$, we can rewrite the two conditions as:

$$\left(\delta + \lambda + \mu\left(\rho\right)\right) \rho = \frac{\sigma}{k}\frac{b}{c}$$

and:

$$\left(\delta + \lambda + \mu\left(\rho\right) + \mu'\left(\rho\right)\rho\right) \rho = \frac{1}{k}\frac{b}{c}.$$

Solving this system for k and σ, we get that:

$$k = F\left(\rho\right) := \frac{1}{\left(\delta + \lambda + \mu\left(\rho\right) + \mu'\left(\rho\right)\rho\right)\rho}\frac{b}{c}$$

and:

$$\sigma = G\left(\rho\right) := \frac{\delta + \lambda + \mu\left(\rho\right)}{\delta + \lambda + \mu\left(\rho\right) + \mu'\left(\rho\right)\rho}.$$

Note that $F\left(\rho\right) > 0$ and $0 < G\left(\rho\right) < 1$, for all ρ.

Now, by differentiation, we can show that:

$$F'\left(\rho\right) = -\frac{\delta + \lambda + \mu\left(\rho\right) + 3\mu'\left(\rho\right)\rho + \mu''\left(\rho\right)\rho^2}{\left(\left(\delta + \lambda + \mu\left(\rho\right) + \mu'\left(\rho\right)\rho\right)\rho\right)^2}\frac{b}{c} < 0$$

and:

$$G'\left(\rho\right) = \frac{\left(\delta + \lambda + \mu\left(\rho\right)\right)\mu'\left(\rho\right)}{\left(\delta + \lambda + \mu\left(\rho\right) + \mu'\left(\rho\right)\rho\right)^2}\left(\frac{\mu'\left(\rho\right)\rho}{\delta + \lambda + \mu\left(\rho\right)} - \frac{\mu''\left(\rho\right)\rho}{\mu'\left(\rho\right)} - 1\right).$$

As $F\left(\rho\right)$ is monotonous, it is inversible, leaving:

$$\rho = F^{-1}\left(k\right),$$

from which we can define:

$$g\left(k\right) = G\left(F^{-1}\left(k\right)\right).$$

Under Assumption 3, we can further show that:

$$\lim_{\rho \to 0} F\left(\rho\right) = \infty,$$

$$\lim_{\rho \to 0} G\left(\rho\right) = 1,$$

$$\lim_{\rho \to 1} F\left(\rho\right) = 0$$

and:

$$\lim_{\rho \to 1} G(\rho) = 0.$$

Also, Assumption 3 implies that:

$$\phi'(\rho) = \frac{\mu'(\rho)}{\mu(\rho)} \left(1 - \frac{\mu'(\rho)\rho}{\mu(\rho)} + \frac{\mu''(\rho)\rho}{\mu'(\rho)} \right) \geq 0.$$

Since:

$$\frac{\mu'(\rho)\rho}{\delta + \lambda + \mu(\rho)} - \frac{\mu''(\rho)\rho}{\mu'(\rho)} - 1 < \frac{\mu'(\rho)\rho}{\mu(\rho)} - \frac{\mu''(\rho)\rho}{\mu'(\rho)} - 1$$

and:

$$\frac{\mu'(\rho)\rho}{\mu(\rho)} - \frac{\mu''(\rho)\rho}{\mu'(\rho)} - 1 \leq 0,$$

we show by transitivity that:

$$G'(\rho) = \frac{(\delta + \lambda + \mu(\rho))\mu'(\rho)}{(\delta + \lambda + \mu(\rho) + \mu'(\rho)\rho)^2} \left(\frac{\mu'(\rho)\rho}{\delta + \lambda + \mu(\rho)} - \frac{\mu''(\rho)\rho}{\mu'(\rho)} - 1 \right) < 0.$$

Finally, we obtain by differentiation:

$$g'(k) = G'\left(F^{-1}(k)\right)\left(F^{-1}\right)'(k) = \frac{G'\left(F^{-1}(k)\right)}{F'\left(F^{-1}(k)\right)} > 0.$$

(ii) Let us compare n^* and n°, which satisfy respectively:

$$\int_0^{n^*} \left(\frac{\omega(i)}{\omega(n^*)} \right)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i = (1 - \sigma) \frac{b}{\delta f}.$$

and:

$$\int_0^{n^\circ} \left(\frac{\omega(i)}{\omega(n^\circ)} \right)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i = \frac{1 - \sigma}{\sigma} \frac{b}{\delta f}.$$

As:

$$\int_0^{n} \left(\frac{\omega(i)}{\omega(n)} \right)^{\frac{\sigma}{1-\sigma}} \mathrm{d}i$$

is increasing in n and (knowing that $0 < \sigma < 1$):

$$\frac{1 - \sigma}{\sigma} \frac{b}{\delta f} > (1 - \sigma) \frac{b}{\delta f},$$

it follows that $n^* < n^\circ$.

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